

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Discrete Mathematics

Subject Code: 4TE04DSM1

Branch: B.Tech (CE,IT)

Semester: 4

Date: 03/05/2017

Time: 02:00 To 05:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
 - (2) Instructions written on main answer book are strictly to be obeyed.
 - (3) Draw neat diagrams and figures (if necessary) at right places.
 - (4) Assume suitable data if needed.
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Q-1 Attempt the following questions:

(14)

- a) Find the least and greatest element in the POSET $\langle N, D \rangle$, if they exist. (01)
- b) Define: Equivalence relation, Comparable element. (02)
- c) State Pigeonhole principle. (01)
- d) Prove that $(ab + ab')a'b' = 0$. (02)
- e) Find the atom and anti-atom of $\langle P(X), \subseteq \rangle$. (01)
- f) State Handshaking theorem and define cycle. (02)
- g) $(Z_{10}, +_{10})$ is cyclic group.- True or False? (01)
- h) Define: Difference of two fuzzy sets. (02)
- i) In how many ways can the letters of the word MONDAY be arranged? How many of them begin with M and end with Y? (02)

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions.

(14)

- a) $\langle L, *, \oplus \rangle$ is a lattice an algebraic system then there exist an order relation \leq on L such that $\langle L, \leq \rangle$ is a lattice as a poset. Where $a * b = \text{glb}\{a, b\}$, $a \oplus b = \text{lub}\{a, b\}$ for $\forall a, b \in L$. (05)
- b) For a lattice $\langle S_{60}, D \rangle$, answer the following questions: (05)
 - i) Find cover of each element and draw the Hasse diagram.
 - ii) Find lower bound, upper bound, greatest lower bound, least upper bound of $A = \{6, 12\}$.
- c) $\langle L, *, \oplus \rangle$ is a modular lattice if and only if $(a * b) \oplus (a * c) = a * (b \oplus (a * c))$. (04)



Q-3 Attempt all questions (14)

- a) Show that $\{1, 5, 7, 11\}$ is a subgroup of (Z_{12}^*, \times_{12}) , where \times_{12} is multiplication modulo 12. (05)
- b) Prove that $\langle P(X), \subseteq \rangle$ is a complemented lattice and also draw the Hasse diagram of it, where $X = \{1, 2, 3\}$. (05)
- c) Show that w is a valid conclusion from the premises $r \vee s, \sim t \Rightarrow \sim r, \sim s, t \Rightarrow w$. (04)

Q-4 Attempt all questions (14)

- a) Let $\langle L, *, \oplus, ', 0, 1 \rangle$ be a complemented lattice and for any $a, b \in L$ then prove that $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b = 1$. (05)
- b) Let $E = \{0, 1, 2, 3\}$, $\underline{A} = \{(0, 0.2), (1, 0.4), (2, 0.6), (3, 0.7)\}$, $\underline{B} = \{(0, 0.4), (1, 0.6), (2, 0.8), (3, 0.8)\}$ then find the following: (05)
 - 1) $\underline{A} \cup \underline{B}$ 2) $\underline{A} \cdot \underline{B}$ 3) $\underline{A} \hat{+} \underline{B}$ 4) $\underline{A} - \underline{B}$ 5) \underline{A}'
- c) Obtain the sum of product canonical form of the Boolean expression in three variables $\alpha(x, y, z) = (x + y)(x + z') + y + z'$. (04)

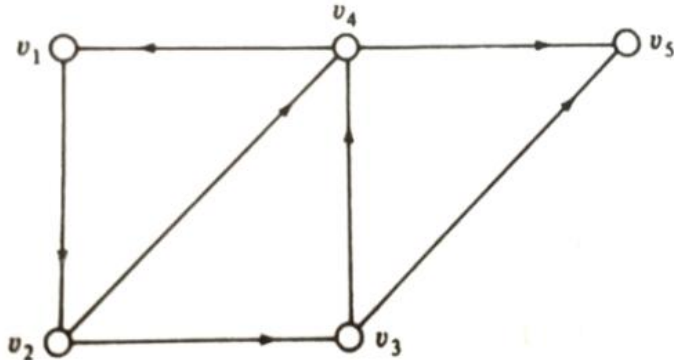
Q-5 Attempt all questions (14)

- a) State and prove Stone's representation theorem. (10)
- b) State Distributive law for fuzzy subsets and prove any one. (04)

Q-6 Attempt all questions (14)

- a) i) Draw the graph represented by given adjacency matrix $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$. (05)

ii) Write the adjacency matrix from the given digraph.



- b) Show that the set $Q \setminus \{1\}$ is an abelian group with respect to the binary operation $a * b = a + b - ab$, for all $a, b \in G$. (05)



c) Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, $n \geq 2$; $a_0 = 1$, $a_1 = 2$. (04)

Q-7 Attempt all questions. (14)

a) Define: Pseudo graph, Cycle, Reachable set, Node base, Level of vertex (05)

b) Prove that $(Z_5, +_5)$ is a group. Moreover check that it is cyclic or not, if it is cyclic then find generators. (05)

c) By using mathematical induction prove that $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$. (04)

Q-8 Attempt all questions. (14)

a) Draw a directed tree from following and also find the representation of binary tree. (05)

$$\left(v_0 \left(v_1 \left(\left(v_2\right)\left(v_3\right)\left(v_4\left(v_5\right)\left(v_6\right)\right)\right)\right)\left(v_7 \left(\left(v_8\left(\left(v_9\right)\left(v_{10}\right)\left(v_{11}\right)\right)\right)\left(v_{12}\right)\right)\right)$$

b) State and prove the Lagrange's theorem. (05)

c) Do as directed: (04)

1) Translate each of the statement into symbols, using quantifiers, predicate symbols.

i) "Every student either can speak Tamil or knows C++."

ii) "Some men are genius."

2) Translate the following in your own words.

$A(x)$: x is a whale, $B(x)$: x is a fish, $C(x)$: x lives in water.

i) $(\exists x)(B(x) \wedge \sim A(x))$

ii) $(\forall x)(A(x) \vee C(x)) \Rightarrow B(x)$

